New Model Free Measurement Based Approach to Engineering Design

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Outline I

Basic foundation of "Measurement Based Approach"

Consider the system of parametrized linear equations

$$\mathbf{A}(\mathbf{p})\mathbf{x} = \mathbf{b}(\mathbf{p})$$

where $\mathbf{A}(\mathbf{p})$ is an $n \times n$ matrix, and $\mathbf{x}$ and $\mathbf{b}(\mathbf{p})$ are $n \times 1$ vectors all with real or complex entries.

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Assuming that $|\mathbf{A}(\mathbf{p})| \neq 0$, there exists a unique solution $\mathbf{x}$ and the i^{th} component x_i of $\mathbf{x}$ is given by

$$x_i(\mathbf{p}) = \frac{|\mathbf{B}_i(\mathbf{p})|}{|\mathbf{A}(\mathbf{p})|}$$

where $\mathbf{B}_i(\mathbf{p})$ is the matrix obtained by replacing the i^{th} column of $\mathbf{A}(\mathbf{p})$ by $\mathbf{b}(\mathbf{p})$.

x represents the system variables such as currents, voltages, displacements, flow rates, and **p** represents a vector of design parameters which appears affinely in $\mathbf{A}(\mathbf{p})$ and $\mathbf{b}(\mathbf{p})$. Thus, we can write

$$\mathbf{A}(\mathbf{p}) := \mathbf{A}_0 + p_1\mathbf{A}_1 + p_2\mathbf{A}_2 + \cdots + p_l\mathbf{A}_l.$$

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Lemma (A Special Case: $\mathbf{p} = p_1$)

Let

$$\mathbf{A}(\mathbf{p}) = \mathbf{A}_0 + p_1\mathbf{A}_1.$$

Then $|\mathbf{A}(\mathbf{p})|$ is a polynomial of degree at most r_1 in p_1 where

$$r_1 = \mathrm{rank}\left[\mathbf{A}_1\right].$$

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Proof

The proof follows easily from the properties of determinants.

Lemma

With

$$\mathbf{A}(\mathbf{p}) := \mathbf{A}_0 + p_1\mathbf{A}_1 + p_2\mathbf{A}_2 + \cdots + p_l\mathbf{A}_l,$$

let

$$r_i = \mathrm{rank}\left[\mathbf{A}_i\right], \quad i = 1, 2, \cdots, l.$$

Then $|\mathbf{A}(\mathbf{p})|$ is a multivariate polynomial in $\mathbf{p}$ of degree r_i or less in p_i, $i = 1, 2, \cdots, l$ and

$$|\mathbf{A}(\mathbf{p})| = \sum_{i_l=0}^{r_l} \cdots \sum_{i_2=0}^{r_2} \sum_{i_1=0}^{r_1} \alpha_{i_1 i_2 \cdots i_l} p_1^{i_1} p_2^{i_2} \cdots p_l^{i_l} =: \alpha(\mathbf{p})$$

$$\left|\mathbf{B}_i(\mathbf{p})\right| = \sum_{i_l=0}^{t_l} \cdots \sum_{i_2=0}^{t_2} \sum_{i_1=0}^{t_1} \beta_{i_1 i_2 \cdots i_l} p_1^{i_1} p_2^{i_2} \cdots p_l^{i_l} =: \beta(\mathbf{p})$$

where $\mathbf{B}_i(\mathbf{p})$ is the matrix obtained by replacing the i^{th} column of $\mathbf{A}(\mathbf{p})$ by the vector $\mathbf{b}(\mathbf{p})$ and

$$t_i = \mathrm{rank}\left[\mathbf{B}_i(\mathbf{p})\right] \quad i = 1, 2, \cdots, l.$$

Theorem (A characterization of parametrized solutions)

Let

$$\mathbf{A}(\mathbf{p})\mathbf{x} = \mathbf{b}(\mathbf{p})$$

where

$$\mathbf{A}(\mathbf{p}) = \mathbf{A}_0 + p_1\mathbf{A}_1 + p_2\mathbf{A}_2 + \cdots + p_l\mathbf{A}_l.$$

Then

$$x_i(\mathbf{p}) = \frac{\beta_i(\mathbf{p})}{\alpha(\mathbf{p})}, \quad i = 1, 2, \cdots, n$$

where $\beta_i(\mathbf{p})$, $i = 1, 2, \cdots, n$ and $\alpha(\mathbf{p})$ are multivariate polynomials in $\mathbf{p}$.

For an unknown model, $(\mathbf{A}(\mathbf{p})$ and $\mathbf{b}(\mathbf{p}))$ are not known. However we assume that the ranks r_i and t_i are known. The coefficient in

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denoted by the vectors α and β are unknown, and the number of unknown coefficients is

$$\mu := \Pi_{i=1}^{l} (r_i + 1) + \Pi_{i=1}^{l} (t_i + 1) - 1.$$

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However, these coefficients can be determined by setting the parameter vector $\mathbf{p}$ to μ different sets of values and solving a set of μ linear equations in the μ unknowns.

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A Measurement Theorem

The function $\mathbf{x}(\mathbf{p})$ can be determined from μ measurements and solution of a system of μ linear equations called the measurement equations in the unknown coefficient vectors α and β.

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We use this fact

to develop our extremal result for the class of linear systems containing real parameters with interval uncertainties and a special type of parameter dependence.

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There exists no p such that $A(p)$ is a singular matrix.

Reason: if $\exists\ p_0 \mid A(p_0)$: singular $\Rightarrow x$: not unique (which is not the case for physical systems).
We define the following sets:

$$\mathcal{P} := \{\mathbf{p}, \mathbf{q}\} = \{p_1, p_2, \ldots, p_l, q_1, q_2, \ldots, q_m\}$$
$$\mathcal{X} := \{x_1, x_2, \ldots, x_n\}$$

Consider the i^{th} element of $\mathcal{X}$, x_i, whose value over a box in the **parameter space** $\mathcal{D}$, where $\mathcal{D} \subset \mathcal{P}$, is to be evaluated.

Case 1: $\mathcal{D} = \{p_1\}$

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Theorem

Suppose $A(p) = A_0 + p_1 A_1$ and $\mathrm{rank}(A_1) = 1$, then the function

$$x_i(\mathbf{p}, \mathbf{q}) = \frac{\beta(\mathbf{p}, \mathbf{q})}{\alpha(\mathbf{p})}, \quad i = 1, 2, \ldots, n$$

can be determined by setting p_1 to 3 different values and measuring the corresponding x_i values.

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Proof.

Since $\mathrm{rank}(A_1) = 1$, and based on the first Lemma:

$$x_i(p_1) = \frac{\tilde{\beta}_0 + \tilde{\beta}_1 p_1}{\tilde{\alpha}_0 + \tilde{\alpha}_1 p_1}$$

If $\tilde{\alpha}_0 = \tilde{\alpha}_1 = 0$, $x_i \to \infty, \forall p_1$, which is not physically possible $\Rightarrow$ we rule out this case.

If $\tilde{\alpha}_1 \neq 0$:

$$x_i(p_1) = \frac{\beta_0 + \beta_1 p_1}{\alpha_0 + p_1} \tag{1}$$

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To find α's and β's:

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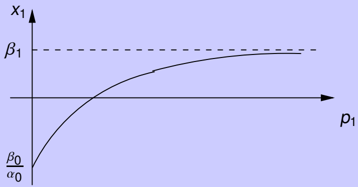
$$\begin{bmatrix} 1 & p_1^1 & -x_i^1 \\ 1 & p_1^2 & -x_i^2 \\ 1 & p_1^3 & -x_i^3 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} x_i^1 p_1^1 \\ x_i^2 p_1^2 \\ x_i^3 p_1^3 \end{bmatrix}$$

Remark:

The function (1) is *monotonic* in p_1.



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Theorem

Suppose

$$A(p) = A_0 + p_1 A_1 \quad \text{and} \quad \mathrm{rank}(A_1) = 1,$$

and p_1 is varying in an interval, $\mathcal{I} = [p_1^-, p_1^+]$, then the extremal values of x_i can be obtained from:

$$\min_{p_1 \in \mathcal{I}} x_i(p_1) = \min\{x_i(p_1^-), x_i(p_1^+)\}$$

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Proof.

The proof follows from the previous Theorem and Remark.

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Theorem

Suppose that

$$A(\mathbf{p}) = A_0 + p_1 A_1 + p_2 A_2 \quad \text{and} \quad \mathrm{rank}(A_1) = \mathrm{rank}(A_2) = 1,$$

then the function $x_i(p_1, p_2)$ can be determined by assigning 7 different sets of values to (p_1, p_2) and measuring the corresponding x_i values.

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Proof.

Since $\mathrm{rank}(A_1) = \mathrm{rank}(A_2) = 1$ and based on the second Lemma:

$$x_i(p_1, p_2) = \frac{\beta_0 + \beta_1 p_1 + \beta_2 p_2 + \beta_3 p_1 p_2}{\alpha_0 + \alpha_1 p_1 + \alpha_2 p_2 + p_1 p_2}$$

which is *monotonic* with respect to p_1 and p_2.

Theorem

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and p_1 and p_2 are varying in a rectangle, $\mathcal{R}$:

$$\mathcal{R} = \left\{ (p_1, p_2) \mid p_1^- \leq p_1 \leq p_1^+,\ p_2^- \leq p_2 \leq p_2^+ \right\}$$

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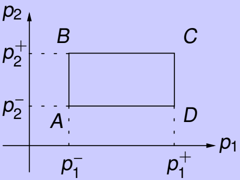

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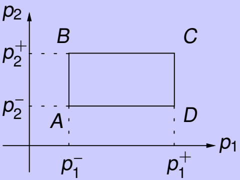

then the extremal values of x_i happen at the vertices of $\mathcal{R}$:

$$\min_{p_1, p_2 \in \mathcal{R}} x_i(p_1, p_2) = \min \{x_i(A), x_i(B), x_i(C), x_i(D)\}$$

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Theorem

If

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$$b(\mathbf{q}) = b_1 q_1 + b_2 q_2 + \cdots + b_m q_m,$$

then $x_i(\mathbf{p}, \mathbf{q})$ can be determined by assigning $2^l(2^m + 1) - 1$ linearly independent sets of values to $(\mathbf{p}, \mathbf{q})$, measuring the corresponding values of x_i and solving a system of measurement equations.

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Theorem

If $\mathrm{rank}(A_i) = 1,\ i = 1, 2, \ldots, l$, and $(\mathbf{p}, \mathbf{q})$ are varying in a box, $\mathcal{B}$:

$$\mathcal{B} = \left\{(\mathbf{p}, \mathbf{q}) \mid p_i^- \le p_i \le p_i^+,\ i = 1, \ldots, l,\ q_j^- \le q_j \le q_j^+,\ j = 1, \ldots, m\right\}$$

with $v := 2^{l+m}$ vertices, labeled $V_1, V_2, \ldots, V_v$, then:

$$\min_{\mathbf{p},\mathbf{q} \in \mathcal{B}} x_i(\mathbf{p}, \mathbf{q}) = \min\{x_i(V_1), x_i(V_2), \ldots, x_i(V_v)\}$$
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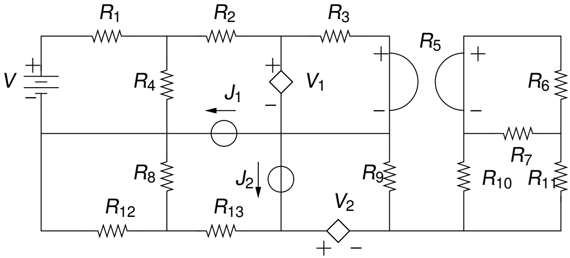
Outline I

Illustrative Examples - Electrical Circuits



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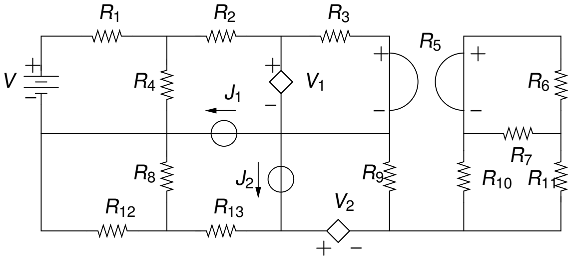

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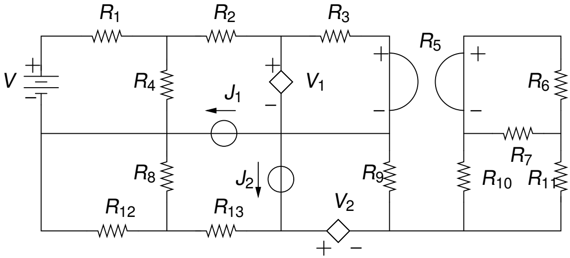

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- $\mathbf{p} = [R_1, R_2, \ldots, R_{13}, K_1, K_2]^T$: vector of circuit parameters

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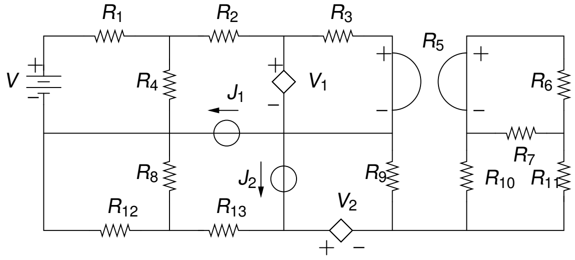

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- $\mathbf{q} = [V, J_1, J_2]^T$: vector of independent sources

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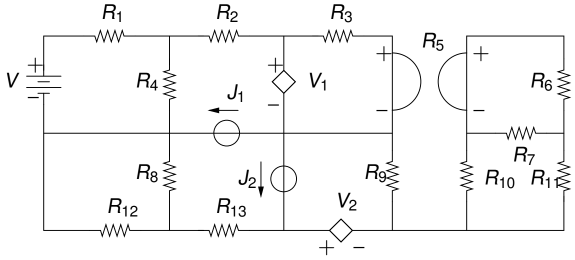

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- $\mathbf{x}$: vector of unknown currents

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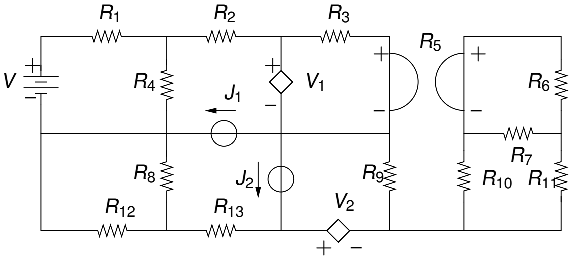

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- $\mathbf{q} = [V, J_1, J_2]^T$: vector of independent sources
- $\mathbf{x}$: vector of unknown currents
- unknown circuit: **p** and **q** are unknown

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Find the extremal values of I_2, if R_1 is varying in the interval $\mathcal{I} = [R_1^-, R_1^+] = [10,\ 30]\ (\Omega)$.

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Solution:

We have $A(R_1) = A_0 + R_1 A_1$, and $\mathrm{rank}(A_1) = 1$, thus the extremal values of I_2 occur at $R_1^- = 10\ (\Omega)$ and $R_1^+ = 30\ (\Omega)$: $\quad I_{2,\min} = 4.7\ (A), \quad I_{2,\max} = 6.3\ (A)$

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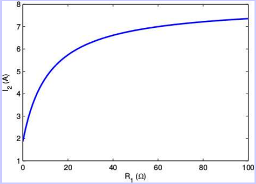
First find the function $I_2(R_1)$ by 3 measurements, and then evaluate I_2 at the vertices of $\mathcal{I}$:

Exp. No.	R_1 (Ω)	I_2 (A)
1	7	4.2
2	18	5.6
3	32	6.4



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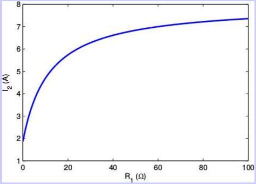

$$I_2(R_1) = \frac{21.9 + 8R_1}{11.7 + R_1}$$

Problem:

Consider the same circuit and suppose that the uncertain parameters are R_1 and R_6 varying in the rectangle $\mathcal{R}$:

$$\mathcal{R} = \{(R_1, R_6) \mid 5 \leq R_1 \leq 15,\ 2 \leq R_6 \leq 5\ (\Omega)\}$$

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Solution:

The power level P_3 can be expressed as:

$$P_3(R_1, R_6) = R_3\ I_3^2(R_1, R_6)$$

but $I_3(R_1, R_6)$ is monotonic in R_1 and R_6, thus our extremal result is valid and we have:

$$P_{3,\min} = 49.4\ (W) \text{ at vertex B} = (5, 5)$$
$$P_{3,\max} = 150\ (W) \text{ at vertex D} = (15, 2)$$

Alternative Approach:

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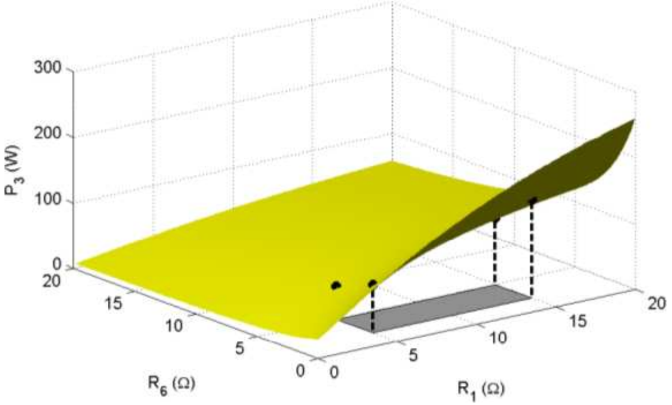
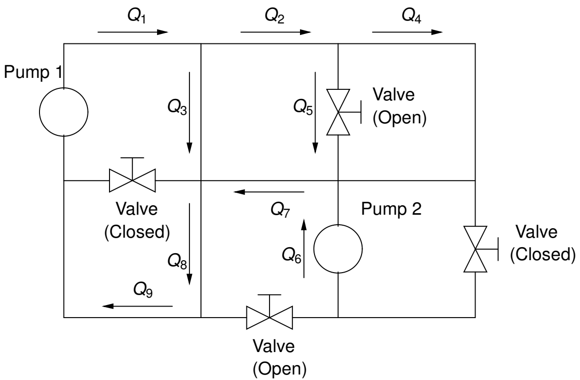


Illustrative Examples - Hydraulic Networks



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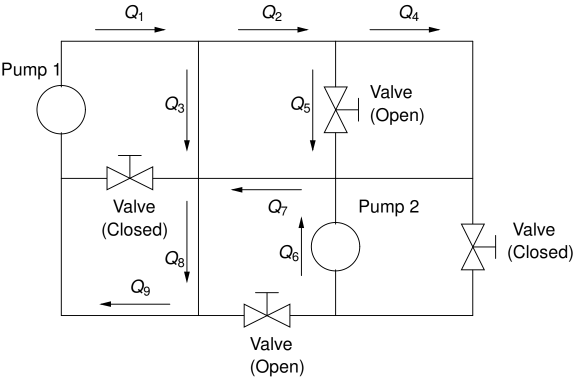

Assumption

Assuming that flows are in the laminar state:

$$\mathbf{A}(\mathbf{p})\mathbf{x} = \mathbf{b}(\mathbf{q})$$

- Pipe resistance: $R = \frac{8\mu L}{\pi r^4}$ where μ is the dynamic viscosity of the fluid, L and r are the length and radius of the pipe

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Observation:

Each pipe resistance appears with rank one dependency in $A(p)$.

Problem:

Suppose that the radii of pipes numbered 2 and 9 are varying in:

$$\mathcal{R} = \{(r_2, r_9) \mid 0.08 \leq r_2 \leq 0.14,\ 0.07 \leq r_9 \leq 0.10\ (m)\}$$

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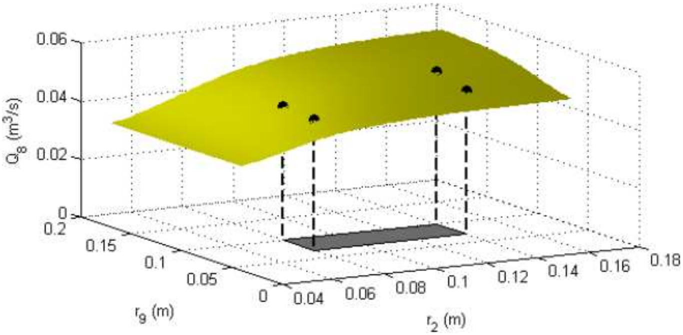
Solution:

Rank one dependency holds, therefore:

$$Q_{8,\min} = 0.045\ (m^3/s) \text{ at vertex A} = (0.08, 0.07)$$
$$Q_{8,\max} = 0.053\ (m^3/s) \text{ at vertex C} = (0.14, 0.10)$$



0.06
0.04
0.02
0
Q_8 (m^3/s)
0.2
0.15
0.1
0.05
0
r_9 (m)
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
r_2 (m)

Outline I

Reliable Measurement-Based System Design

Objective

Proposing a approach to the design of fault-tolerant systems.

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Key Feature

- does not require a mathematical model of the plant and is strictly based on measurement data.
- determines the design element values to preserve acceptable system performance under predetermined failures of fault prone elements.

Fault Tolerant System Design: Single Failure

Consider the following system

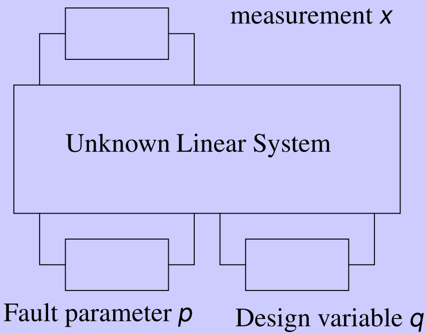

Figure : Fault Tolerant System Design (Single Failure)

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- For example, $p = p_0$ (some fixed value) for normal state and $p = 0$ (or ∞) if a failure occurs.
- The functional dependency of the performance measurement x on the fault prone parameter p and the design variable q can be written as

$$x(p,q) = \frac{\alpha_0 + \alpha_1 p + \alpha_2 q + \alpha_3 pq}{\beta_0 + \beta_1 p + \beta_2 q + pq}.$$

Fault Tolerant System Design: Single Failure (Continue)

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- The coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2$ can be determined by conducting 7 experiments with 7 different sets of values of the parameters p and q, and taking the corresponding measurements x.

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$$x_{\min} \le \min_{p \in [p^1, p^2, p^3]} x(p,q), \qquad x_{\max} \ge \max_{p \in [p^1, p^2, p^3]} x(p,q).$$

Fault Tolerant System Design: Two Failures

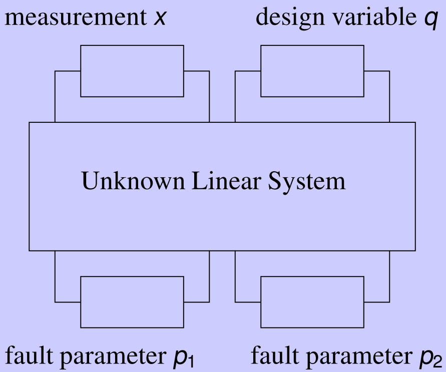

Figure : Fault tolerant system design (two failure prone elements)

Fault Tolerant System Design: Two Failures (Continue)

The functional dependency of the performance variable x on the two fault prone parameter $\mathbf{p} = [p_1, p_2]$ and the design parameter q can be written as

$$x(\mathbf{p}, q) = \frac{\beta(\mathbf{p}, q)}{\alpha(\mathbf{p}, q)} = \frac{\alpha_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 q + \alpha_4 p_1 p_2 + \alpha_5 p_1 q + \alpha_6 p_2 q + \alpha_7 p_1 p_2 q}{\beta_0 + \beta_1 p_1 + \beta_2 p_2 + \beta_3 q + \beta_4 p_1 p_2 + \beta_5 p_1 q + \beta_6 p_2 q + p_1 p_2 q}.$$

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- Clearly, 15 experiments with 15 different sets of values of $(\mathbf{p}, q)$ and the corresponding measurements of x suffice to determine all the coefficients representing the above functional dependency.

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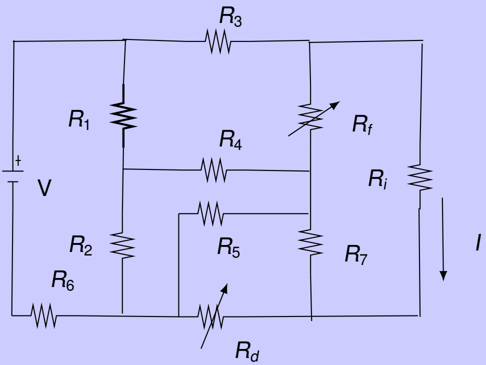
- Suppose that the fault prone parameter $\mathbf{p}$ undergoes states $\bar{\mathbf{p}} := \{\mathbf{p}^k, k = 1, 2, \cdots\}$ representing all possible failure states.
- Then the fault tolerant design is obtained by selecting the design variable q or a range of q values such that for the given acceptable tolerance $x \in [x_{\min}, x_{\max}]$, we achieve

$$x_{\min} \leq \min_{\mathbf{p} \in \bar{\mathbf{p}}} x(\mathbf{p}, q), \qquad x_{\max} \geq \max_{\mathbf{p} \in \bar{\mathbf{p}}} x(\mathbf{p}, q).$$

Outline I

Example 1 (A single failure)



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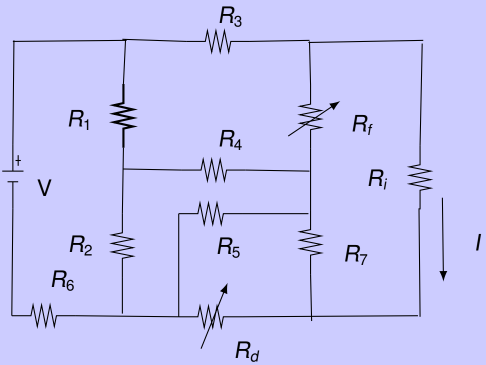

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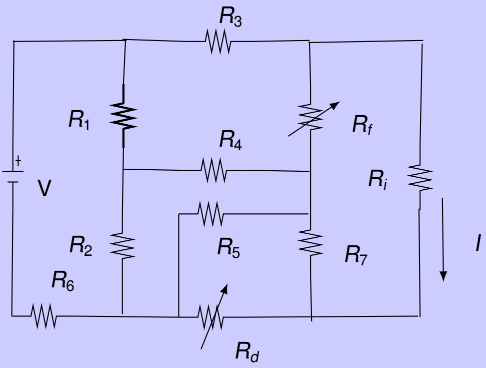

- Suppose that resistor R_f is most vulnerable to faults.
- The goal is to design R_d so that the current through R_i denoted as I should be in the range:

$$[I_{\min}, I_{\max}] = [3.2A, 6.8A].$$

Design:

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We find the functional dependency of current I on the resistors R_d and R_f is given by,

$$I(R_d, R_f) = \frac{\alpha_0 + \alpha_1 R_d + \alpha_2 R_f + \alpha_3 R_d R_f}{\beta_0 + \beta_1 R_d + \beta_2 R_f + R_d R_f}.$$

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To determine $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1$ and β_2, seven experiments should be conducted by setting 7 different values for the resistors R_d, and R_f, and measuring the corresponding current values I. The experiments are conducted on the Simulink model of the circuit.

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Note that in the design process, it is assumed that the model is not known. By setting 7 different values for the resistors R_d, and R_f the corresponding currents I are measured.

Measurements for experiments with R_d, R_f.

Exp.No	$R_d(\Omega)$	$R_f(\Omega)$	$I(R_d, R_f)(A)$
1	1	1	4.52
2	7	2	3.07
3	12	7	4.09
4	20	13	4.41
5	35	24	4.65
6	67	43	4.79
7	90	58	4.85

Then solve

$$\begin{bmatrix} 1 & R_{d1} & R_{f1} & R_{d1}R_{f1} & -I_1 & -I_1R_{d1} & -I_1R_{f1} \\ 1 & R_{d2} & R_{f2} & R_{d2}R_{f2} & -I_2 & -I_2R_{d2} & -I_2R_{f2} \\ 1 & R_{d3} & R_{f3} & R_{d3}R_{f3} & -I_3 & -I_3R_{d3} & -I_3R_{f3} \\ 1 & R_{d4} & R_{f4} & R_{d4}R_{f4} & -I_4 & -I_4R_{d4} & -I_4R_{f4} \\ 1 & R_{d5} & R_{f5} & R_{d5}R_{f5} & -I_5 & -I_5R_{d5} & -I_5R_{f5} \\ 1 & R_{d6} & R_{f6} & R_{d6}R_{f6} & -I_6 & -I_6R_{d6} & -I_6R_{f6} \\ 1 & R_{d7} & R_{f7} & R_{d7}R_{f7} & -I_7 & -I_7R_{d7} & -I_7R_{f7} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} I_1R_{d1}R_{f1} \\ I_2R_{d2}R_{f2} \\ I_3R_{d3}R_{f3} \\ I_4R_{d4}R_{f4} \\ I_5R_{d5}R_{f5} \\ I_6R_{d6}R_{f6} \\ I_7R_{d7}R_{f7} \end{bmatrix}.$$

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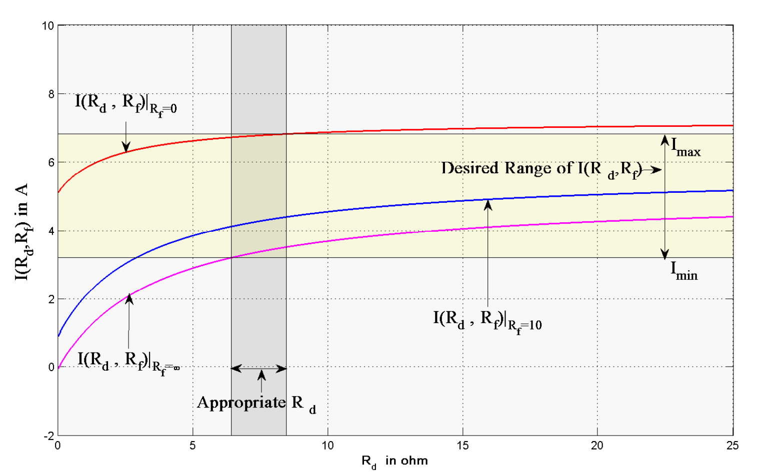
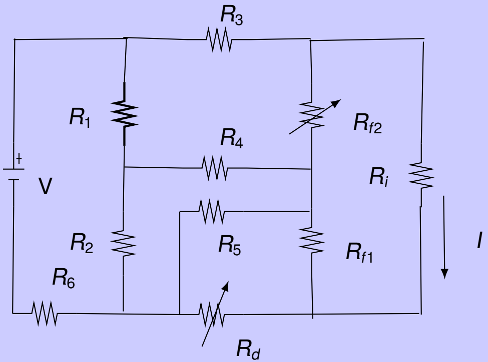
The functional dependency of current I on the resistors R_d, and R_f is found to be

$$I(R_d, R_f) = \frac{42.6594 - 0.1964R_d + 30.5111R_f + 5.0467R_dR_f}{8.3847 + 3.6402R_d + 4.2363R_f + R_dR_f}.$$

I vs. R_d, for fixed R_f.



Example 2 (Two failures)



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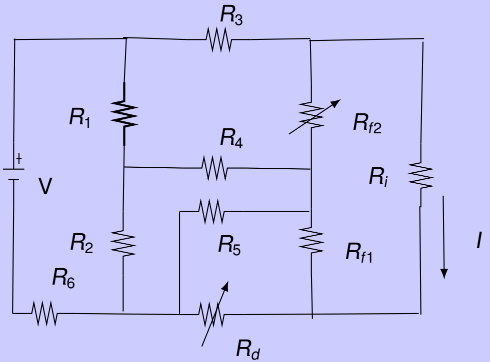

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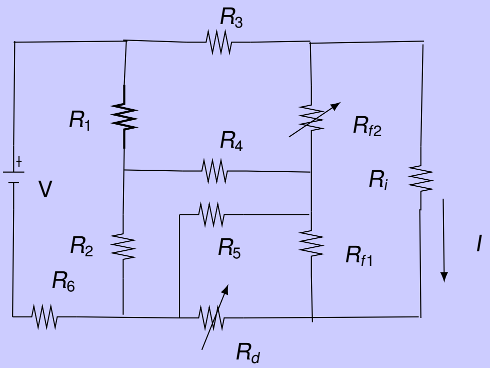

- Resistors R_{f1} and R_{f2} are most vulnerable to faults.
- We want to design R_d so that the current through R_i denoted as I should be in the range:

$$[I_{\min}, I_{\max}] = [0.5A, 4A].$$

The functional dependency of current I on the resistors R_d, R_{f1}, and R_{f2}. The relation is given by

$$I(R_d, R_{f1}, R_{f2}) = \frac{\alpha_0 + \alpha_1 R_d + \alpha_2 R_{f1} + \alpha_3 R_{f2} + \alpha_4 R_d R_{f1} + \alpha_5 R_{f1} R_{f2} + \alpha_6 R_{f2} R_d + \alpha_7 R_d R_{f1} R_{f2}}{\beta_0 + \beta_1 R_d + \beta_2 R_{f1} + \beta_3 R_{f2} + \beta_4 R_d R_{f1} + \beta_5 R_{f1} R_{f2} + \beta_6 R_{f2} R_d + R_d R_{f1} R_{f2}}$$

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To determine α_0, α_1, α_2, α_3, α_4, α_5, α_6, α_7, β_0, β_1,β_2, β_3, β_4, β_5, and β_6, fifteen experiments should be conducted by setting 15 different sets of values for the resistors R_d,R_{f1}, and R_{f2}, and measuring the corresponding current I.

$I(R_d, R_{f1}, R_{f2})$ measurements for experiments with R_d, R_{f1}, R_{f2}.

Exp.No	$R_d(\Omega)$	$R_{f1}(\Omega)$	$R_{f2}(\Omega)$	$I(R_d, R_{f1}, R_{f2})(A)$
1	1	1	1	3.53
2	2	3	4	3.62
3	7	7	7	4.35
4	10	9	12	3.99
5	13	12	17	3.68
6	18	16	20	3.54
7	21	20	25	3.24
8	24	28	31	2.85
9	29	35	35	2.65
10	34	42	43	2.38
11	39	51	56	2.06
12	43	67	67	1.77
13	51	75	70	1.71
14	58	83	79	1.59
15	75	90	85	1.53

$$I(R_d, R_{f1}, R_{f2}) = \frac{7.5 - 2.1R_d - 0.9R_{f1} + 1.28R_{f2} + 0.03R_dR_{f1} + 0.08R_{f1}R_{f2} + 0.11R_{f2}R_d - 0.0}{2 - 0.5R_d - 0.4R_{f1} + 0.5R_{f2} + 0.01R_dR_{f1} + 0.006R_{f1}R_{f2} + 0.02R_{f2}R_d + 0.0}$$

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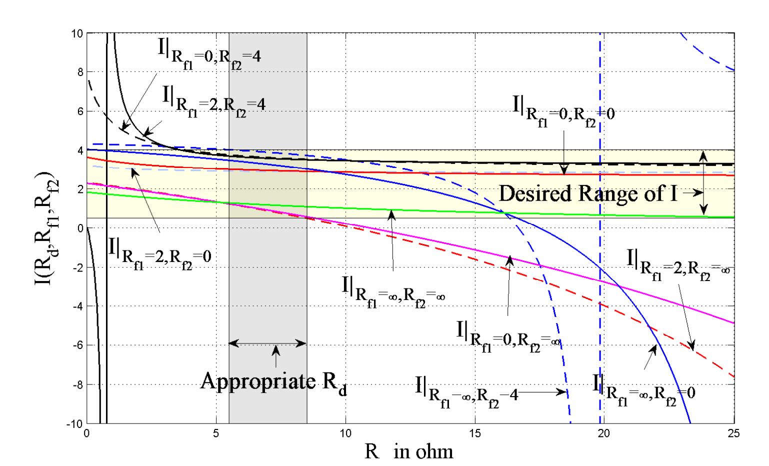
For different failure conditions listed in Table and for $R_d \in [0, 25]$, current $I(R_d, R_{f1}, R_{f2})$ is calculated using the expression above.

Table : Single or Two resistor Failure Conditions Considered in Design

Fault Condition	R_{f1}	R_{f2}
1	Short	Normal
2	Open	Normal
3	Normal	Short
4	Normal	Open
5	Short	Short
6	Short	Open
7	Open	Short
8	Open	Open
9	Normal	Normal

I vs. R_d, for fixed R_{f1}, and R_{f2}.



- Then the appropriate range of R_d for fault tolerance has to be selected so that the conditions below are satisfied for all the fault conditions considered in Table for fault conditions.

$$I(R_{d,\min}, R_{f1}, R_{f2}) \in [0.5A, 4A], \qquad I(R_{d,\max}, R_{f1}, R_{f2}) \in [0.5A, 4A]$$

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- From the figure, it is found that $R_{d,\min} = 5.5\Omega$ and $R_{d,\max} = 8.5\Omega$ is the appropriate range of R_d. When R_d is set to any value within the above range, the current I is maintained within the desired range for any faults at R_{f1} and R_{f2}.

Outline I

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- If the parameters appear with rank one dependency in the characteristic matrix of the system, then the parametrized solutions will be monotonic in these parameters.
- This monotonic characteristic is used to show that the extremal values of the parametrized solutions over a box in the parameter space occur at the vertices of the box.

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- This is illustrated by examples of design of fault tolerant electrical circuits. Research is ongoing for extending this theory to fault tolerant system and controller design for general linear time invariant systems.

Thank you